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CAREER SPECTRA DEHRADUN

MATHEMATICAL PHYSICS

Assignment – Complex Analysis

"CSIR-NET/JRF JUNE-2021"

For –

- ☞ CSIR-NET/JRF
- ☞ GATE / JEST
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PART-A (MODULUS & ARGUMENT-CUBE ROOTS OF UNITY)

1. Multiplying a complex number z by $(1-\sqrt{3}i)$ rotates the radius vector of z by an angle of.

(a) 60^0 clockwise
(b) 30^0 anticlockwise
(c) 30^0 clockwise
(d) 60^0 anticlockwise

2. If $|z_1+z_2| = |z_1-z_2|$, then the phase difference between z_1 and z_2 is.

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

3. The complex number 'z' for which arg (z+1) = $\frac{\pi}{6}$ and arg (z-1)= $2\frac{\pi}{3}$, is

(a) $\frac{\sqrt{3}}{2}+i\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}-i\frac{1}{2}$ (c) $\frac{1}{2}-i\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$

4. If z ≠ 0 is a complex number such that arg (z) = $\frac{\pi}{4}$, then

(a) Re (z^2) = lm (z^2)
(b) Re $(z^2) = 0$
(c) lm $(z^2) = 0$
(d) None of these

5. If complex number z_1, z_2 and origin form an equilateral triangle, then which of following relation is TRUE?

(a) $z_1^2+z_2^2+z_1z_2=0$
(b) $z_1^2+z_2^2-z_1z_2=0$
(c) $z_1^2+z_2^2=0$
(d) $z_1^2+z_2^2-2z_1z_2=0$

6. The locus of the point z defined by the equation arg (z-4) = $\frac{\pi}{4}$, is

(a) Circle (b) Parabola (c) Ellipse (d) Straight line

7. If 1, ω, ω^2 are the complex cube roots of unity, then the value of the following expression:

$(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$………… to 2n factors.

(a) 2n (b) 2^{2n} (c) 0 (d) 1

8. If 1, ω, ω^2 are the complex cube roots of unity, then $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ will be.

(a) 4 (b) 8 (c) 16 (d) 32

9. If x + iy = $\left(i\sqrt{3}-1\right)^{100}$. then the co-ordinate of point P (x,y) will be.

(a) $\left(2^{90}, 2^{99}\sqrt{3}\right)$
(b) $\left(2^{90}, -2^{99}\sqrt{3}\right)$
(c) $\left(-2^{99}, 2^{99}\sqrt{3}\right)$
(d) None of these

PART-B (COMPLEX FUNCTION & CAUCHY-REAMANN EQUATIONS)

1. Examine the continuity of the following functions :

(i) $f(z) = \begin{cases} \frac{Re(z^3)}{|z|^2} & for\ z \neq 0 \\ 0 & for\ z = 0 \end{cases}$ at z = 0

(ii) $f(z) = \begin{cases} \frac{z^2+1}{z+i} & for\ z \neq -i \\ 0 & for\ z = -i \end{cases}$ at z = -i

(iii) $f(z) = \begin{cases} \frac{x^3y^5(x+iy)}{x^4+y^4} & for\ z \neq 0 \\ 0 & for\ z = 0 \end{cases}$ at z = 0

(iv) $f(z) = \begin{cases} \exp\left(-\frac{1}{z^2}\right) & for\ z \neq 0 \\ 0 & for\ z = 0 \end{cases}$ at z = 0

[Answer : (i) continuous. (ii) not continuous, (iii) continuous, (iv) continuous,]

2. Compute the following limit

(i) $\lim_{Z\to\infty} \frac{iz^3-iz+1}{(2z-3i)(z+i)^2}$

(ii) $\lim_{Z\to\infty} \frac{z^3}{Re(z^3)-lm\,(z^3)}$

(iii) $\lim_{Z\to\infty} \frac{1-\cos z}{\sin z^2}$

(iv) $\left[\sqrt{z-2i} - \sqrt{z-i}\ \right]$

(v) $\lim_{Z\to\infty} \frac{(Re\,z-lm\,z)^2}{|z|^2}$

(vi) $\lim_{Z\to\infty} \frac{z^4-1}{z+i}$

[Answer: (i) $\frac{i}{2}$, (ii) Limit does not exist, (iii) $\frac{1}{2}$, (iv)0, (v) Limit does not exist, (vi) 4i]

3. Let $u(r,\theta) = -r^3 \sin 3\theta$ be the real part of an analytic function $f(z)$ of the complex variable z = r. $e^{i\theta}$, the imaginary part of $f(z)$, will be.

(a) $r^3 \cos 3\theta + C$ (b) $-r^3 \cos 3\theta + C$

(c) $-ir^3 \cos 3\theta + C$ (d) $ir^3 \cos 3\theta + C$

4. Let $u(x,y) = x^2 - y^2 - 2x$ be the real part of an analytic function f (z) of the complex variable z = x +iy. The imaginary part of the analytic function, will be.

(a) $2xy - 2y + c$ (b) $x^2 y - 2y + c$

(c) $x^2 y - y^2 + c$ (d) $2xy - y^2 + c$

5. Let $v(r,\theta) = r^2 \cos 2\theta - 2r\cos\theta + 2$ be the imaginary part of an analytic function f(z) of the complex variable z = $r.e^{i\theta}$. The real part of f (z), will be

(a) $r^2 \sin 2\theta - 2r \sin\theta + C$ (b) $r^2 \sin 2\theta + 2r \sin\theta + C$
(c) $-r^2 \sin 2\theta + 2r \sin\theta + C$ (d) $-r^2 \sin 2\theta - 2r \sin\theta + C$

6. Let $u(x, y) = -x^2 + xy + y^2$ be the real part of an analytic function f(z) of the complex variable z = x +iy. Then f(z) can be expressed as.
(a) $f(z) = \frac{1}{2}(1 + i)z^2$ (b) $f(z) = -\frac{1}{2}(2 + i)z^2$
(c) $f(z) = \frac{1}{2}(2 - i)z^2$ (d) $f(z) = \frac{1}{2}(1 - i)z^2$

7. If $f(z) = \frac{1}{2}\ln(x^2 + y^2) + i\, tan^{-1}\left(\frac{ax}{y}\right)$ be an analytic function, then a is equal to.
(a) -1 (b) 1 (c) -2 (d) 2

8. Given: $f(z) = x^2 + Py^2 - 2xy + i\,(Qx^2 - y^2 + 2xy)$ is analytic in nature. The value of P and Q will be.
(a) P = 1, Q = 1 (b) P = -1, Q = 1
(c) P =1, Q = -1 (d) P = -1, Q = -1

PART-C (MILNE THOMSON METHOD & ANALYTIC FUNCTION)

1. Find the complex analytic function $f(z)$ for which either the real part or the imaginary part is given as following:
(i) $v = e^x(x\cos y - y\sin y)$ (ii) $u - v = (x - y)(x^2 + 4xy + y^2)$
(iii) $v = tan^{-1}\left(\frac{y}{x}\right)$
(iv) $2u + v = e^{2x}[(2x + y)\cos 2y + (x - 2y)\sin 2y]$
(v) $u(x, y) = 2x + y^3 - 3x^2y$
Ans.
(i) $ize^z + C$, (ii) $-iz^3 + C$,
(iii) ln z + C, (iv) $ze^{2z} + C$, (v) $2z+iz^3 + C$]

2. The real part of the complex analytic function $f(z)$ is given by $u(x, y) = Ax + By$. If can be written as $f(z) = Mz + C$ (where C is constant), then the value of M is.
(a) A+ iB (b) A –iB (c) A+B (d) A – B

3. Which of the following function is NOT analytic in the entire complex argand plane ?
(a) $f(z) = |z|^2$ (b) $f(z) = \bar{z}$
(c) $f(z) = z(Re\ z)$ (d) $f(z) = \cos z$

4. Which of the following function is analytic at the origin in the complex argand plane?
(a) $f(z) = i|z|^2$ (b) $f(z) = z(lm\ z)$
(c) $f(z) = z^3$ (d) $f(z) = \frac{z+2i}{1-iz}$

5. Which of the following **CANNOT** be a real part of a complex analytic function f(z) of the complex variable z = x +iy?
(a) $\frac{1}{2}\ln(x^2+y^2)$ (b) sin x cosh y
(c) $e^{-2xy}\sin(x^2-y^2)$ (d) $x^2 + y^2$

6. If the function $v(x,y) = e^{ax}\sin h(by)$ corresponding to the imaginary part of the complex analytic function $f(z) = u(x,y) + iv(x,y)$, then which of the following relation is **CORRECT?**
(a) $b = \pm a$ (b) $b = \pm ia$
(c) $b = \pm i2\pi a$ (d) $b^2 = \pm a^2$

7. Which of the following is/are NOT a complex analytic function of complex variable $z = x + iy$?
(a) $f(z) = (x^2 - y^2 + 2ixy)^7(x+iy)^{17}$
(b) $f(z) = (x^2 - y^2 + 2ixy)^{12}(x-iy)^4$
(c) $f(z) = (x + iy - 5)^{13}$
(d) $f(z) = (2x + iy - 5)^{19}$

PART-D (POWER & TAYLOR SERIES EXPANSION)

1. Find the radius and region of convergence of the power series expansion of the following functions:
(i) $f(z) = \frac{1}{(z-3)(z+2)}$ about z = 1 (ii) $f(z) = \frac{1}{(z-3)(z-4)}$ about z = 1
(iii) $f(z) = \frac{1}{z^2+(1+2i)z+2i}$ about z = 0 (iv) $f(z) = \ln(2+iz)$ about z = 1
(v) $f(z) = \sinh z$ about z = $\frac{\pi i}{2}$ (vi) $f(z) = \sin(2z + z^2)$ about z = -1
(vii) $f(z) = cosech\ (z)$ about z =i $\frac{\pi}{2}$
(viii) $f(z) = \ln\left(\frac{1+z}{1-z}\right)$ about z = 0
[ANSWER: (i) R = 2,$|z-1| < 2$, (ii) R =1, $|z-2| < 1$ (iii) R =1, $|z| < 1$,
(iv) R =1, $|z - i| < 1$, (v) R = ∞, $\left|z - \frac{\pi i}{2}\right| < \infty$, (vi) R =∞, $|z + 1| < \infty$,
(vii) R = $\frac{\pi}{4}$, $\left|z - i\frac{\pi}{4}\right| < \frac{\pi}{4}$, (viii) $R = 1, |z| < 1$**]**

2. The taylor series expansion of $f(z) = \cos z$ about z = $\frac{\pi}{3}$ will be.
(a) $f(z) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(z - \frac{\pi}{3}\right) + \frac{1}{4}\left(z - \frac{\pi}{3}\right)^2 + \ldots\ldots$
(b) $f(z) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(z - \frac{\pi}{3}\right) - \frac{1}{4}\left(z - \frac{\pi}{3}\right)^2 + \ldots\ldots$
(c) $f(z) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(z - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(z - \frac{\pi}{3}\right)^2 + \ldots\ldots$

(d) $f(z)=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(z-\frac{\pi}{3}\right)+\frac{\sqrt{3}}{4}\left(z-\frac{\pi}{3}\right)^2+\ldots\ldots.$

3. Suppose a complex function f(z) such that $f(1)=1, f'(1)=1, f''(1)=1$ and all other higher derivatives of $f(z)$ at z = 1, are zero. The value of $f(z)=1/3$ will be.
(a) 1/3 (b) 4/9 (c) 5/9 (d) 7/9

4. Expand of following function into Laurrent series for the regions:
(i) $0<|z|<1$ (ii) $1<|z|<2$ (iii) $2<|z|<\infty$

$$f(z)=\frac{1}{z(z-1)(z-2)}$$

[Answer: (i) $\frac{1}{2z}+\sum_{n\geq 0}\left(1-\frac{1}{2^{n+2}}\right)z^n$, (ii) $-\frac{1}{2z}-\sum_{n\geq 2}\frac{1}{z^n}-\sum_{n\geq 0}\frac{z^n}{2^{n+2}}$
(iii) $\frac{1}{z^3}+\frac{3}{z^4}+\ldots\ldots$]

5. The coefficient of $(x-1)^3$ of Taylor series expansion $f(x)=(x-1)e^x$ about x =1, will be
(a) e/6 (b) e/2 (c) -e/2 (d) -e/6

6. Using the taylor series expansion of the function $f(x) = \sin \pi x$ about x = ½, one can approximate $\sin \pi\left(\frac{1}{2}+\frac{1}{10}\right)$ as (upto 4 decimal places)
(a) 0.9317 (b) 0.9434 (c) 0.9511 (d) 0.9632

7. The Taylor series expansion of the function $f(x)=\cos x.\ln(1-x)$ about x = 0, will be.
(a) $-x-\frac{x^2}{2}+\frac{x^3}{6}+\ldots$ (b) $x-\frac{x^2}{2}+\frac{x^3}{6}+\ldots$
(c) $x-\frac{x^2}{2}+\frac{x^3}{6}-\ldots$ (d) $-x-\frac{x^2}{2}-\frac{x^3}{6}-\ldots$

8. The coefficient of $(x-1)^4$ of Taylor series expansion of $f(x)=\frac{1}{x^2}$ about x = 1, will be
(a) -5 (b) 5 (c) -4 (d) 4

9. The Taylor series expansion of the function $f(z)=z^3-10z^2+6$ about z = 3 will be.
(a) $57-33(z-3)+(z-3)^2-(z-3)^3$
(b) $-57-33(z-3)+(z-3)^2-(z-3)^3$
(c) $-57-33(z-3)-(z-3)^2+(z-3)^3$
(d) $-57-33(z-3)-(z-3)^2-(z-3)^3$

10. Taylor series expansion of the function $f(z)=z^4e^{-3z^2}$, about z = 0 will be.
(a) $\sum_{n=0}^{\infty}\frac{(-3)^n z^{2n+4}}{n!}$ (b) $\sum_{n=1}^{\infty}\frac{(-3)^n z^{2n+4}}{n!}$

(c) $\sum_{n=1}^{\infty}\frac{(-3)^n z^{2n+4}}{(2n)!}$ (d) $\sum_{n=0}^{\infty}\frac{(-3)^n z^{2n+4}}{(2n)!}$

11. For which of the following functions, the Laurent series about the origin has largest region of convergence?

(a) $\frac{1}{z^2-2z}$ (b) $\frac{e^{z-1}}{z}$ (c) $\frac{1}{(z+1)(z-2)}$ (d) $\frac{1}{z(z-1)}$

12. Expand the following complex functions in Laurent series:

(i) $f(z)=\frac{1}{z(z+2)^3}$ about z = -2 (ii) $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1

(iii) $f(z)=\frac{1}{(z+1)(z+2)}$ about z = -2 (iv) $f(z)=\frac{z-\sin z}{z^3}$ about z = 0

(v) $f(z)=\cos\left(\frac{z}{1-z}\right)$ about z = 1 (vi) $f(z)=\exp\left(\frac{z}{z-2}\right)$ about z = 2

(vii) $f(z)=(z+3)\sin\left(\frac{1}{z-2}\right)$ about z = 2

(viii) $f(z)=\frac{1}{z^2}\sinh\left(\frac{1}{z}\right)$ about z = 0

(ix) $f(z)=\frac{1}{z^2+(3i-1)z-3i}$ about z = 1

(x) $f(z)=\sin\left[\frac{z^2-6z}{(z-3)^2}\right]$ about z = 3

[**ANSWER :** (i) $-\frac{1}{2(z+2)^3}-\frac{1}{4(z+2)^2}-\frac{1}{8(z+2)}-\frac{1}{16}-\frac{1}{32}(z+2)-$..........

(ii) $\frac{e^2}{2(z-1)^3}+\frac{2e^2}{(z-1)^2}+\frac{2e^2}{(z-1)}+\frac{4e^2}{3}+\frac{2e^2}{3}(z-1)-$..........

(iii) $\frac{2}{z+2}+1+(z+2)+(z+2)^2+\ldots\ldots$ (iv) $\frac{1}{3i}-\frac{z^2}{5!}+\frac{z^4}{7!}$

(v) $\sum_{n=0}^{\infty}\frac{(-1)^n\cos 1}{(2n)!(z-1)^{2n}}-\sum_{n=0}^{\infty}\frac{(-1)^n\sin 1}{(2n+1)!(z-1)^{2n+1}}$ (vi) $e\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{2}{z-2}\right)^n$

(vii) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left[\frac{1}{(z-2)^{2n}}+\frac{1}{(z-2)^{2n+1}}\right]$ (viii) $\sum_{n=0}^{\infty}\frac{1}{(2n-1)!z^{2n+1}}$

(ix) $\frac{(1-3i)}{10}\left[\frac{1}{z-1}-\sum_{n=0}^{\infty}\frac{(-1)^n(z-1)^n}{(1+3i)^{n+1}}\right]$

(x) $(\sin l)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(\frac{3}{z-3}\right)^{4n}-(\cos 1)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)!}\left(\frac{3}{z-3}\right)^{4n-2}$

13. In the Laurent series expansion of the function f(z)= $\frac{1}{(z-1)(z-2)}$ in the annular region $1<|z|<2$, the ratio of the coefficient of z^n and $\frac{1}{z^n}$ will be.

(a) $\frac{1}{2}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2^{n+1}}$ (d) $-\frac{1}{2^n}$

PART-E (SINGULAR POINTS & CALCULATION OF RESIDUES)

1. Determine the singular point and corresponding residues of following complex functions:

(i) $f(z) = \frac{1-2z}{z(z-1)(z-2)}$ (ii) $f(z) = \frac{z^2}{z^2+a^2}$

(iii) $f(z) = \frac{ze^{iz}}{z^4+a^4}$ (iv) $f(z) = \frac{z^2}{(z+1)^2+(z-2)^2}$

(v) $f(z) = \frac{z}{\sin z}$ (vi) $f(z) = \frac{z^2}{z^2(z^2+9)}$ at (0,-3)

(vii) $f(z) = z^2 sin\frac{1}{z}$ (viii) $f(z) = \cot z$

(ix) $f(z) = \sec z$ (x) $f(z) = \coth(z)$

(xi) $f(z) = \frac{e^{iz}+\cos z}{(z-\pi)^4}$ (xii) $f(z) = e^{z-\frac{1}{2}}$

(xiii) $f(z) = \frac{\cos 2z}{(z+1)^2}$ (xiv) $f(z) = \frac{\exp(imz)}{z(z^2+a^2)^2}$

(xv) $f(z) = \frac{1}{ln^2 z}$

[Answer: **(i)** z – 0,1,2, Residue = $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$, **(ii)** z = $\pm ia$, Residue = $\left(\frac{1}{2}ia, -\frac{1}{2}ia\right)$, **(iii)** z = $\pm a, \pm ia$. Residue = $\left(\frac{1}{4a^2}e^{ia}, \frac{1}{4a^2}e^{-ia}, -\frac{1}{4a^2}e^{-a}, -\frac{1}{4a^2}e^{a}\right)$,

(iv) z = -1,2, Residue = $\left(\frac{5}{9}, \frac{4}{9}\right)$, **(v)** z = $n\pi$, Residue = $\frac{n\pi}{(-1)^n}$,

(vi) $-\frac{e^{-3i}}{54}$

(vii) z = 0, Residue = $-\frac{1}{6}$, **(viii)** $z = n\pi$, Residue = 1,

(ix) $z = (2n+1)\frac{\pi}{2}, Residue = (-1)^{k+1}$, **(x)** $z = in\pi$, Residue = 1

(xi) $z = n\pi$, Residue = $\frac{i}{6}$

(xii) z = 0, Residue = $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(n!)(n+1)!}$

(xiii) z = -1, Residue = 2 (sin 2),

(xiv) z = 0, $\pm ia$, Residue = $\frac{1}{a^4}, -\frac{(2+ma)e^{-ma}}{4a^4}, -\frac{(2-ma)e^{ma}}{4a^4}$,

(xv) z= 1, Residue =1]

2. If the function f(z) is a polynomial of order n, then at z = ∞, the function has a.
(a)Classical singularity (b)Removable singularity
(c)Simple Pole (d) Pole of order n

3. If the function f(z) has a pole order n at z =a, the order of the pole of $f^{(m)}$ (z) at this point will be.
(a)n – m (b)m – n (c)m + n (d) 0 if m = n

4. If the function *f* (z) has a pole order n at z = a, the order of the pole of *f'*(z) at this point will be.
(a)n – 2 (b)n -1 (c)n (d) n +1

5. If the function f (z) has a pole order n at z = a, the order of the pole of $\frac{f''(z)}{f'(z)}$ at this point will be.

(a)n – 2 (b)n – 1 (c)n (d) 1

6. At z = 0, the function f (z) = tan $\left(\frac{1}{z}\right)$ has.

(a)An isolated simple pole (b)An isolated pole of order 2
(c)A non-isolated essential singularity
(d) An isolated removable singularity

7. The function f(z) = $\frac{1}{1-e^{z}}$, has a singularity at z = 2πi of the following nature:

(a)Pole of order 1
(b)Removable isolated singularity
(c)Isolated essential singularity
(d) Non-isolated essential singularity

8. The function f(z) = (z-3) sin $\left(\frac{1}{z+2}\right)$ has singularity at z = -2 of the following nature:

(a)Pole of order 1 (b)Removable isolated singularity
(c)Isolated essential singularity (d) Non-isolated essential singularity

9. Discuss the nature of singularity regarding the following complex functions:

(i) $f(z) = \frac{z}{(z^2+4)^2}$

(ii) $f(z) = sec\left(\frac{1}{2}\right)$ (iii) $f(z) = \frac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$

(iv) $f(z) = \frac{1}{\sin \pi z^2}$ (v) $f(z) = sin\left(\frac{1}{1-z}\right)$

(vi) $f(z) = \frac{\cot z}{z}$ (vii) $f(z) = \frac{z}{1+z^4}$

(viii) $f(z) = \frac{\sin z}{(z-\pi)^2}$ (ix) $f(z) = (z-\pi)\exp\left(\frac{1}{2}\right)\cot z$

(x) $f(z) = \frac{(z^2-1)(z-2)^3}{[\sin \pi z]^3}$

Answer:

(i) z = $\pm 2i$ is a pole of order 2,
(ii) z = $\frac{2}{(2n+1)\pi}$(n= $0, \pm 1, \pm 2 \ldots.$) are simple pole.
(iii) z = 1 is a pole of order 3, z = -$\frac{2}{3}$ is pole of order 2,
(iv) z = 0 is a pole of order 2, z = $\pm\sqrt{n}(n = \pm 1 \ldots..)$ are simple poles,
(v) z= 1 is an essential singular point,
(vi) z = 0 is a pole of order 2, $z = n\pi(n = \pm 1, \ldots..)$ are simple pole,
(vii) z = $e^{i\frac{\pi}{4}}, e^{i\frac{3\pi}{4}}, e^{i\frac{5\pi}{4}}, e^{i\frac{7\pi}{4}}$ are simple poles.

(viii) z = π is a simple pole,
(ix) z = 0 is essential singular point, z = π is removable singular point, others are simple poles (x) z = ± 1 are poles of order 2, z = 2 is removable singular point, others are simple poles].

10. At z = 0, the residue of the function $f(z) = z^n \sin\left(\frac{1}{z}\right)$[n is an integer] will be non-zero if.

(a)Z < 0 (b) n > 0 and odd
(c)n ≥ 0 and even (d) Only for n = 0

11. Which of the following statements is CORRECT for the function $f(z) = \frac{1}{z(e^z-1)}$?

(a)z = 0 is a simple pole and the corresponding residue is ½.
(b)z = 0 is a simple pole and the corresponding residue is -½.
(c)z = 0 is a double pole and the corresponding residue is ½.
(d) z = 0 is a double pole and the corresponding residue is -½.

12. The function $f(z) = (z-3)^n \sin\left(\frac{1}{z-3}\right)$ has a residue of $\frac{1}{120}$ at the point z = 3. The value of 'n' is.

(a)2 (b)4 (c)5 (d) 6

13. Which of the following statement is CORRECT for the function $f(z) = \frac{1}{z\sin z}$?

(a) z = 0 is a simple pole and the corresponding residue is ½.
(b) z = 0 is a simple pole and the corresponding residue is 0.
(c) z = 0 is a double pole and the corresponding residue is ½.
(d) z = 0 is a double pole and the corresponding residue is 0.

14. Find the residue of the following functions at z = ∞:

(i) $f(z) = \frac{z^4+z^2}{z^3}$ (ii) $f(z) = \frac{z}{e^{-z^2}+1}$ (iii) (i) $f(z) = z^3 \cos\left(\frac{1}{z}\right)$

[Answer: (i) -1 (ii) 0, (iii)-1/24]

PART-F (APPLICATION OF CAUCHY RESIDUE THEOREM)

1. The contribution of the point z = π/2 in evaluation of $\oint_C \frac{\tan z}{z} dz$ (where C is a circle $|z| = 2$) is.

(a) 0 (b) $-4e^{i\pi/2}$ (c) $4e^{i\pi/2}$ (d) $-2/\pi$

2. Around which of the following curves the integral $\oint_C \frac{z-1}{z^2+1} dz$ is vanishing ?

(a) $|z+i| = 1$ (b) $|z-i| = 1$

(c) $|z-1| = 1$ (d) $2x^2 + (y+1)^2 = 1$

3. Which of the following contour integral vanishes around circle C : $|z| = 2$?

(a) $\oint_C \frac{\sin z}{z} dz$ (b) $\oint_C \frac{e^z}{z-1} dz$

(c) $\oint_C \frac{z^2}{z+i} dz$ (d) $\oint_C \frac{z^2-1}{z^3-z^2+9z-9} dz$

4. The contour integral $\oint_C z^2(z-2)^{n-1} dz$ around $|z| = 3$ vanishes if the integer n is such that.

(a) n < -2 (b) n > -2 (c) n = -2 (d) none of these

5. The value of the contour integral $\oint_C \frac{z^3}{(z-2)^2} dz$ over $|z| = 1$, is

(a) $24\pi i$ (b) $12\pi i$ (c) $6\pi i$ (d) 0

6. The value of the integral $\oint_C \frac{3z^3+z+1}{(z^2-1)(z+3)} dz$ around the curve C :$|z| = 2$ (where, 'C' is traverse in the clockwise direction) is equal to.

(a) $\frac{3\pi i}{4}$ (b) $-\frac{3\pi i}{4}$ (c) $\frac{\pi i}{4}$ (d) $-\frac{\pi i}{4}$

7. The value of the integral $\oint_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is a circle $|z| = 2$ in the argand plane, described in the positive sense is equal to.

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/5$

8. The value of the integral $\oint_C \frac{1}{z^3-z^4} dz$, where C is a circle $|z| = 1/2$ in the argand plane, described in the positive sense, is

(a) $2\pi i$ (b) $-2\pi i$ (c) πi (d) 0

9. The value of the integral $\oint_C \frac{e^z}{\sin z} dz$, where C is the positively traversed rectangle with corners at $-\frac{\pi}{2} - i, -\frac{\pi}{2} + 2i$, and $\frac{5\pi}{2} + 2i$, will be.

(a) $2\pi i\,(1 - e^{\pi} - e^{2\pi})$ (b) $2\pi i\,(1 + e^{\pi} + e^{2\pi})$

(c) $2\pi i\,(1 + e^{\pi} - e^{2\pi})$ (d) $2\pi i\,(1 - e^{\pi} + e^{2\pi})$

10. The value of the integral $\oint_C \frac{z^2-1}{z^2-5iz-4} dz$ where C :$|z-4i| = 2$ (oriented clockwise) is.

(a) $\frac{4\pi}{3}$ (b) $-\frac{4\pi}{3}$ (c) $-\frac{34\pi}{3}$ (d) $\frac{34\pi}{3}$

11. The value of the integral $\oint_C \frac{e^z-1}{z(z-1)(z-3i)^2}dz$ around the curve C : $|z| = 2$ (where 'C' is traversed in the clockwise direction) is equal to.
(a) π (e -1) (b) - π (e -1) (c) πe (d) –πe

12. The value of the integral $\oint_C (z+1)\cot\left(\frac{z}{2}\right)dz$, where C is a circle $|z| = 1$ in the complex argand plane given below described in the negative sense, is
(a) 2πi (b) -2πi (c) 4πi (d) -4πi

13. Evaluate the following integrals:
(i) $\oint_C \frac{\cos z}{z^{2n+1}}dz$;[C: $|z| = 1$] (ii) $\oint_C \frac{\cos(\pi z^2)dz}{(z-1)(z-2)}$; C: $|z| = 3$
(iii) $\oint_C \frac{\sin h\,(3z)}{\left(z-\frac{\pi i}{4}\right)^3}dz$;C: square bounded by x = ±2, y = ±2
(iv) $\oint_C \frac{4z^2-4z+1}{(z-2)(z^2+4)}dz$; C: circle $|z| = 1$
(v) $\oint_C \frac{1}{\sinh z}dz$; C: circle $|z| = 4$
(vi) $\oint_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$; [C: circle $|z| = 2(x \le 0)$]
(vii) $\oint_C e^{-1/z}\sin\left(\frac{1}{z}\right)dz$; C: circle of $|z| = 1$
(viii) $\oint_C \frac{sin\,z}{z^4}dz$; C: circle $|z| = 2$ (ix) $\oint_C \frac{1}{z^4+1}dz$; C: circle of $|z-1| = 1$
(x) $\oint_C \frac{2+3sin\,\pi z}{z(z-1)^2}dz$; C: square having vertices at 3+3i, 3-3i, -3+3i, -3-3i
(xi) $\oint_C \frac{sin\,z}{z^2(z^2-1)}\exp\left[\frac{1}{(z-1)^2}\right]dz$; C: $\left|z+\frac{1}{2}\right| = 1$
(xii)) $\oint_C \tan\pi\,z\,dz$; C : circle of $|z| = 2$,

Ans.
(i) $\frac{2\pi i(-1)^n}{(2n)!}$, (ii) 4πi (iii) $-\frac{9\pi}{\sqrt{2}}$, (iv) 0,
(v) $-2\pi i$ (vi) $-\frac{3}{2}\pi i$ (vii) $2\pi i$ (viii) $-\frac{\pi i}{3}$
(ix) $-\frac{\pi i}{\sqrt{2}}$ (x) - $6\pi^2$i (xi) $\pi i\left[e^{1/4}\sin 1 - 2e\right]$
(xii) -8i]

PART-G (IMPROPER INTEGRAL)

1. Evaluate the following integral:
(i) $\int_0^{2\pi}\frac{sin^2\theta}{5-4\cos\theta}d\theta$ (ii) $\int_0^{2\pi}\frac{cos^2 3\theta}{5-4\cos 2\theta}d\theta$
(iii) $\int_0^{2\pi}\frac{d\theta}{1-2m\cos\theta+m^2}(m^2<1)$ (iv) $\int_0^{2\pi}\frac{sin^2\theta-2\cos\theta}{2+\cos\theta}d\theta$
(v) $\int_0^{2\pi}\frac{d}{a^2+sin^2\theta}d\theta$

Answer:

(i) $\frac{\pi}{4}$, (ii) $\frac{3\pi}{4}$ (iii) $\frac{2\pi}{1-m^2}$ (iv) $\frac{2\pi}{\sqrt{3}}$,
(v) $\frac{\pi}{\sqrt{1+a^2}}$

2. The value of the integral $\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)d\theta$ will be
(a) 0 (b) 2π (c) π (d) $\frac{\pi}{2}$

3. The value of the integral $\int_0^{2\pi} e^{-\cos\theta}\cos(\sin\theta+n\theta)d\theta$ will be.
(a) $\frac{2\pi}{(n+1)!}$ (b) $\frac{2\pi}{n!}$ (c) $\frac{\pi}{n!}(-1)^n$ (d) $\frac{2\pi}{n!}(-1)^n$

4. The value of the integral $\int_0^{2\pi}\frac{\sin 3\theta}{5-3\cos\theta}d\theta$ will be
(a) 0 (b) $\frac{\pi}{n!}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

5. Evaluate the following integral:
(i) $\int_0^{2\pi}\frac{1}{a+b\cos\theta}d\theta\ (a>|b|, a>0)$
(ii) $\int_0^{2\pi}\frac{1}{a+b\sin\theta}d\theta\ (a>|b|, a>0)$
(iii) $\int_0^{2\pi}\frac{1}{(a+b\cos\theta)^2}d\theta\ (a>|b|, a>0)$
(iv) $\int_0^{2\pi}\frac{1}{(a+b\cos\theta)^2}d\theta\ (a>|b|, a>0)$
(v) $\int_0^{2\pi}\frac{1}{\sqrt{2}-\cos\theta}d\theta$
(vi) $\int_0^{2\pi}\frac{1}{(5+4\cos\theta)^2}d\theta$

Answer:
(i) $\frac{2\pi}{\sqrt{a^2-b^2}}$ (ii) $\frac{2\pi}{\sqrt{a^2-b^2}}$ (iii) $\frac{2\pi a}{(a^2-b^2)^{3/2}}$
(iv) $\frac{2\pi a}{(a^2-b^2)^{3/2}}$ (v) 2π (vii) $\frac{10\pi}{27}$]

6. Evaluate the following integrals:
(i) $\int_{-\infty}^{\infty}\frac{\cos ax}{x^2+b^2}dx(a,b>0)$ (ii) $\int_{-\infty}^{\infty}\frac{\sin ax}{x^2+b^2}dx(a,b>0)$
(iii) $\int_{-\infty}^{\infty}\frac{\cos ax}{(x^2+b^2)^2}dx(a,b>0)$
(iv) $\int_{-\infty}^{\infty}\frac{\cos 2x}{(x^2+b^2)(x^2+b^2)}dx(a,b>0)$
(v) $\int_{-\infty}^{\infty}\frac{\sin ax}{x(x^2+b^2)^2}dx(a,b>0)$
(vi) $\int_{-\infty}^{\infty}\frac{\sin mx}{x}dx(m=+ve\ integer)$
(vii) $\int_{-\infty}^{\infty}\frac{\sin mx}{(x^4+a^4)^2}dx(a,m>0)$

Answer:

(i) $\frac{\pi}{b}e^{-ab}$, (ii) 0, (iii) $\frac{\pi(ab+1)}{2b^3}e^{-ab}$,

(iv) $\frac{\pi}{(a^2-b^2)}\left(\frac{e^{-2b}}{b}-\frac{e^{-2b}}{a}\right)$, (v) $\frac{\pi}{b^2}(1-e^{-ab})$,

(vi) π, (vii) $x\exp\left(-\frac{ma}{\sqrt{2}}\right)\cos\left(\frac{ma}{\sqrt{2}}\right)$

7. Evaluate the following integrals:

(i) $\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)}$ (ii) $\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2}$ (iii) $\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^3}$

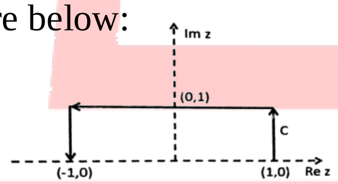
(iv) $\int_{-\infty}^{\infty}\frac{dx}{x^4+a^4}$ (v) $\int_{-\infty}^{\infty}\frac{dx}{(x^4+a^4)^2}$

(vi) $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)}dx(a,b>0)$ (vii) $\int_{-\infty}^{\infty}\frac{1}{(x^2+a^2)(x^2+b^2)}dx(a,b>0)$

[Answer: (i) $\frac{\pi}{a}$, (ii) $\frac{\pi}{2a^3}$ (iii) $\frac{3\pi}{8a^5}$ (iv) $\frac{\pi}{\sqrt{2}a^3}$ (v) $\frac{3\pi}{4\sqrt{2}a^3}$ (vi) $\frac{\pi}{a+b}$, (vii) $\frac{\pi}{ab(a+b)}$]

CSIR PREVIOUS YEAR QUESTIONS

1. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: **[CSIR JUNE-2011]**



(a) $\frac{5}{e}+e$ (b) e - $\frac{5}{e}$

(c) $\frac{5}{e}-e$ (d) - $\frac{5}{e}-e$

2. Which of the following is an analytic function of the complex variable z = x + iy in the domain $|z|<2$? **[CSIR JUNE-2011]**

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4\ (7-x-iy)^3$

(c) $(1-2x-iy)^4\ (3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

3. The first few terms in the Taylor series expansion of the function f(x) = sinx around x = $\frac{\pi}{4}$ are. **[CSIR JUNE-2011]**

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots\right]$

(d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right]$

4. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \leq |z| \leq 2$ and around z =1 is. **[CSIR JUNE-2012]**

(a) $\frac{1}{2}[1+z+z^2+\cdots]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\cdots\right]$

(b) $\frac{1}{1-z}-z-(1-z)^2+(1-z)^3+\cdots$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+\cdots\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+\cdots\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+\cdots$

5. The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}cos\left(\frac{\pi t}{2R}\right)dt$. **[CSIR JUNE-2012]**

(a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

6. Let $u(x,y) = x + \frac{1}{2}(x^2-y^2)$ be the real part of an analytic function *f*(z) of the complex variable z = x + iy. The imaginary part of *f*(z) is. **[CSIR JUNE-2012]**

(a) y + xy (b) xy (c) y (d) $y^2 - x^2$

7. The Taylor series expansion of the function ln (cosh x), where x is real, about point x = 0 starts with the following terms: **[CSIR DEC-2012]**

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+\cdots$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+\cdots$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+\cdots$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+\cdots$

8. The value of the integral $\oint_C \frac{z^3}{z^2-5z+6}dz$, where C is closed contour defined by the equation. $2|z|-5=0$, traversed in the anti-clockwise direction is. **[CSIR DEC-2012]**

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

9. With z = x+ iy, which of the following cannot be the real part of a complex analytic function of z = x+iy? **[CSIR JUNE-2013]**

(a) $(x+iy-8)^3(4+x^2-y^2+2ixy)^7$ (b) $(x+iy)^7(1-x-iy)^3$

(c) $(x^2-y^2+2ixy-3)^5$ (d) $(1-x+iy)^4(2+x+iy)^6$

10. Which of the following function cannot be the real part of a complex analytic function of z = x +iy? **[CSIR DEC-2013]**

(a) x^2y (b) x^2-y^2 (b) x^3-3xy^2 (d) $3x^2y - y - y^3$

11. Given that the integral $\int_0^{\infty}\frac{dx}{y^2+x^2}=\frac{\pi}{2y}$, the value of $\int_0^{\infty}\frac{dx}{(y^2+x^2)^2}$ is. **[CSIR DEC-2013]**

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

12. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{sin^2 z}$ is. **[CSIR JUNE-2014]**

(a) ∞ (b) 2πi (c) 0 (d) πi

13. The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3}$dx is. **[CSIR DEC-2014]**

(a) -2π (b) –π (c) π (d) 2π

14. The Laurent series expansion of the function *f*(z) = $e^z + e^{1/z}$ about z = 0 is. **[CSIR DEC-2014]**

(a) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}\ for\ all\ |z| < \propto$

(b) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}\ only\ if\ 0 < |x| < 1$

(c) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}\ for\ all\ 0 < |z| < \infty$

(d) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$, only if $|z| < 1$

15. Consider the function $f(z) = \frac{1}{z} ln(1-z)$ of a complex variable z = $re^{i\theta}$ $(r \geq 0, -\propto \theta < \infty)$. The singularities of *f*(z) are as follows: **[CSIR DEC-2014]**

(a) Branches points at z =1 and z = ∞; and a pole at z = 0 only for $0 \leq \theta < 2\pi$

(b) Branches points at z =1 and z = ∞; and a pole at z = 0 for all θ other than $0 \leq \theta < 2\pi$

(c) Branches points at z =1 and z = ∞; and a pole at z = 0 for all θ

(d) Branches points at z =0, z = 1 and z = ∞

16. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$ is. **[CSIR JUNE-2015]**

(a) $\pi/\sqrt{2}$ (b) π/2 (c) $\sqrt{2}\pi$ (d) 2π

17. The function $\frac{z}{sin\pi\ z^2}$ of a complex variable z has. **[CSIR DEC-2015]**

(a) A simple pole at 0 and poles of order 2 at z = $\pm\sqrt{n}$ for n = 1,2,3…….

(b) A simple pole at 0 and poles of order at z = $\pm\sqrt{n}$ and z = $\pm i\sqrt{n}$ for n = 1,2,3…….

(c) Poles of order 2 at z = $\sqrt{n}$ for n = 0,1,2,3…….

(d) Poles of order 2 at z = $\pm n$ for n = 0,1,2,3…….

18. The radius of convergence of the Taylor series expansion of the function $\frac{1}{\cosh(x)}$ around x=0, is **[CSIR JUNE-2016]**

(a) ∞ (b) π (c) π/2 (d) 1

19. The value of the contour integral. **[CSIR JUNE-2016]**

$$\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$$

Around the unit circle C traversed in the anti-clockwise direction is.

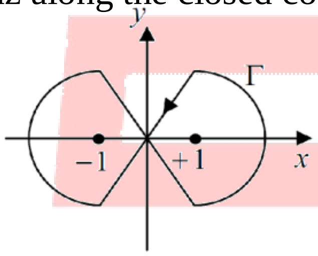
(a) 0 (b) 2 (c) $-\frac{8}{\sqrt{3}}$ (d) $-tanh\left(\frac{1}{2}\right)$

20. Let $u(x,y) = e^{ax}\cos(by)$ the real part of a function $f(z) = u(x,y) + iv(x,y)$ of the complex variable z = x +iy, where a, b are real constant and $a \neq 0$. The function $f(z)$ is complex analytic everywhere in the complex plane if and only if. **[CSIR JUNE-2017]**

(a) $b = 0$ (b) $b = \pm a$

(c) $b = \pm 2\pi a$ (d) $b = a \pm 2\pi$

21. The integral $\oint_\Gamma \frac{ze^{i\pi z/2}}{z^2-1}$ dz along the closed contour Γ shown in the figure is. **[CSIR JUNE-2017]**



(a) 0 (b) 2π (c) -2π (d) 4πi

22. Consider the real function $f(x) = 1/(x^2+4)$. The Taylor expansion of $f(x)$ about x = 0 converges. **[CSIR DEC-2017]**

(a) For all value of x (b) For all values of x except x = ±2

(c) In the region -2 < x < 2 (d) For x > 2 and x < -2

23. What is the value of a for which $f(x,y) = 2x + 3(x^2 - y^2) + 2i(3xy + ay)$ is an analytic function of complex variable z = x+iy. **[CSIR JUNE-2018]**

(a) 1 (b) 0 (c) 3 (d) 2

24. The value of the integral $\oint_C \frac{dz}{z}\frac{\tanh 2z}{\sin \pi z}$, where C is a circle of radius $\frac{\pi}{2}$. traversed counter-clockwise, with centre at z = 0, is **[CSIR DEC-2018]**

(a) 4 (b) 4i (c) 2i (d) 0

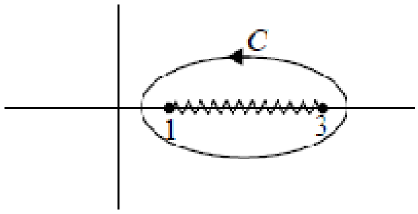
25. The integral I = $\oint_C e^z dz$ is evaluated form the point (-1,0) to (1,0) along the contour C, which is an arc of the parabola y = x^2 -1, as shown in the figure.
The value of I is. **[CSIR DEC-2018]**

(a) 0 (b) 2 sinh 1 (c) e^{2i} sinh 1 (d) $e + e^{-1}$

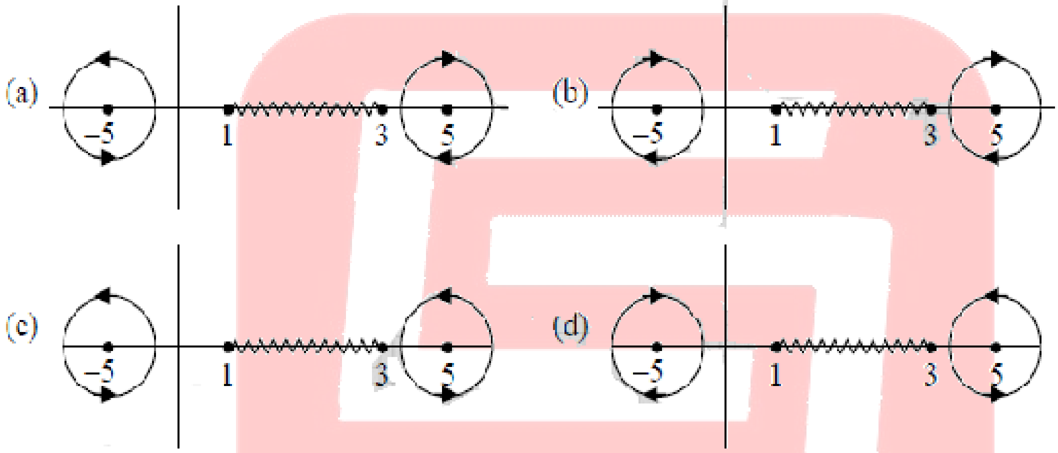
26. The contour C of the following integral. **[CSIR DEC-2018]**

$$\oint_C dz \frac{\sqrt{(z-1)(z-3)}}{(z^2-25)^3}$$

In the complex z plane is shown in the figure below.



This integral is equivalent to an integral along the contours



27. The value of the definite integral $\int_0^{\pi} \frac{d\theta}{5+4\cos\theta}$ is. **[CSIR JUNE-2019]**

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{3}$

28. Let C be the circle of radius $\pi/4$, centered at z = $\frac{1}{4}$ in the complex z-plane that is traversed counter-clockwise. The value of the contour integral $\oint_C \frac{z^2}{sin^2 4z} dz$ is. **[CSIR DEC-2019]**

(a) 0 (b) $\frac{i\pi^2}{4}$ (c) $\frac{i\pi^2}{16}$ (d) $\frac{i\pi}{4}$

29. A function of a complex variable 'z' is defined by the integral f(z) = $\oint_\Gamma \frac{\omega^2-2}{\omega-z} d\omega$. Where Γ is a circular contour of radius 3, centred at origin running counter-clockwise in the w-plane. The value of the function at z = (2 - i) is. **[CSIR-NOV-2020]**

(a) 0 (b) 1 – 4i (c) $8\pi + 2\pi i$ (d) $-\frac{2}{\pi} - \frac{i}{2\pi}$

GATE PREVIOUS YEAR QUESTIONS

30. The value of the integral $\int_C z^{10} dz$, where C is the unit circle with the origin as the centre is: **[GATE-2001]**
(a) 0 (b) z^{11} / 11
(c) 2 π iz^{11} /11 (d) 1/11

31. The value of the residue of $\frac{\sin z}{z^6}$ *is*. **[GATE-2001]**
(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) - $\frac{2\pi i}{5!}$

32. If a function *f*(z) = *u*(x,y)+iv(x,y) of the complex variable z = x+iy, where x,y,u and v are real, is analytic in a domain D of z, then which of the following is true? **[GATE-2002]**
(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

33. The value of the integral $\int_C dz/c^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is: **[GATE-2003]**
(a) 0 (b) 2πi (c) 4πi (d) -4πi

34. The inverse of the complex number $\frac{3+4i}{3-4i}$ is: **[GATE-2004]**
(a) $\frac{7}{25} + i\frac{24}{25}$ (b) $-\frac{7}{25} + i\frac{24}{25}$
(c) $\frac{7}{25} - i\frac{24}{25}$ (d) - $\frac{7}{25} - i\frac{24}{25}$

35. The value of $\oint_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is: **[GATE-2004]**
(a) π for a = 2 (b) zero foe a = $\frac{1}{2}$
(c) 4π for a = 2 (d) $\frac{\pi}{2}$ for a = $\frac{1}{2}$

36. The value of the integral $\int_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z| = 4$, is: **[GATE-2005]**
(a) 0 (b) πi (c) 2 πi (d) 4 πi

37. All solutions of the equation e^z = -3 are. **[GATE-2005]**
(a) $in\pi\ ln3, n = \pm1, \pm2.....$ (b) ln 3+i(2n+1)π, n=0,$\pm1, +2$
(c) ln 3+i2nπ, n=0,$\pm1, +2$ ... (d) i3nπ, n= $\pm1, +2$ ...

38. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}$dz, where C is a circle defined by $|z| = 3$, is: **[GATE-2006]**

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$ (c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

39. The contour integral $\oint \frac{dz}{z^2+a^2}$ is to be evaluated on a circle of radius 2a centered at the origin. It will have contributions only from the points. **[GATE-2006]**
(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) *ia and –ia*
(c) *ia*, -*ia*, $\frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

40. If I = $\oint_C ln\ z\ dz$, where C is the unit circle taken anticlockwise and lnz is the principal branch of the logarithmic function, which of the following is correct? **[GATE-2008]**
(a) I = 0 by residue theorem. (b) I is not defined since, lnz is branch cut.
(c) I ≠ 0 (d) $\oint_C ln\,(z^2)dz = 2I$

41. The value of $\int_{-i}^{i}\pi(z+1)dz$ is. **[GATE-2008]**
(a) 0 (b) 2πi (c) -2πi (d) (–1+2i)π

42. The value of the integral $\int_C \frac{e^z}{z^2-3z+2}dz$, where the contour C is the circle $|z| = \frac{3}{2}$ is. **[GATE-2009]**
(a) 2πie (b) πie (c) -2πie (d) –πie

43. The value of the integral $\oint_C \frac{e^z sinz}{z^2}dz$, where the contour C is the unit circle: $|z-2| = 1, is$ **[GATE-2010]**
(a) 2πi (b) 4πi (c) πi (d) 0

44. For the complex function, *f*(z) = $\frac{e^{\sqrt{z}}-e^{-\sqrt{z}}}{\sin(\sqrt{z})}$, which of the following statement is correct? **[GATE-2010]**
(a) z = 0 is a branch point. (b) z = 0 is a pole of order one
(c) z = 0 is a removable singularity (d) z =0 is an essential singularity

Common data for Q.45 & Q.46-

Consider a function *f*(z) = $\frac{zsinz}{(z-\pi)^2}$ of a complex variable z.

45. Which of the following statements is TRUE for the function *f*(z)? **[GATE-2011]**
(a) *f*(z) is analytic everywhere in the complex plane.
(b) *f*(z) has a zero at z =π
(c) *f*(z) has a pole of order 2 at z = π
(d) *f*(z) has a simple pole at z = π.

46. Consider a counter clockwise circular contour $|z| = 1$ about the origin. The integral $\oint f(z)\,dz$ over this contour is:
(a) –iπ (b) zero (c) iπ (d) 2iπ

47. The value of the integral $\oint_C e^{1/z}dz$, using the contour C of circle with radius $|z| = 1$, is.
[GATE-2012]
(a) 0 (b) 1-2πi (c) 1+2πi (d) 2πi

48. For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole z =1 ……………………………. **[GATE-2013]**
(Your answer should be an integer)

49. The value of the integral $\oint_C \frac{z^2}{e^z+1}dz$, where C is the circle $|z| = 4$, is
[GATE-2014]
(a) 2πi (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

50. Consider w = f(z) = u(x,y)+iv(x,y) to be an analytic function in a domain D. which one of the following options is NOT correct? **[GATE-2015]**
(a) u(x,y) satisfies Laplace equation in D
(b) v(x,y) satisfies Laplace equation in D
(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D.
(d) f(z) can be Taylor expanded in D

51. Consider a complex function $f(z) = \frac{1}{z\left(z+\frac{1}{2}\right)\cos(z\pi)}$. Which one of the following statement is correct? **[GATE-2015]**
(a) f(z) has simple poles at z = 0 and z = $-\frac{1}{2}$
(b) f(z) has a second order pole at z = $-\frac{1}{2}$
(c) f(z) has infinite number of second order poles
(d) f(z) has all simple poles

52. Which of the following is an analytic function of z everywhere in the complex plane? **[GATE-2016]**
(a) z^2 (b) (b) $(z^*)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

53. The contour integral $\oint \frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane circle is equal to ……….(up to two decimal places). Ans = π **[GATE-2017]**

54. The imaginary part of an analytic complex function is v (x,y) = 2xy +3y. The real part of the function is zero at the origin. The value of the real part of the function at 1+ i is………………(up to two decimal places). Ans = 3 **[GATE-2017]**

55. The absolute value of the integral.

$$\int \frac{5z^3+3z^2}{z^2-4}$$

Over the circle $|z - 1.5| = 1$ in complex plane, is …………. (up to two decimal places). Ans = 81.64 **[GATE-2018]**

56. The pole of the function $f(z) = \cot z$ at z = 0 is. **[GATE-2019]**
(a)A removable pole (b)An essential singularity
(c)A simple pole (d)A second order pole

57. The value of the integral $\int_{-\infty}^{\infty} \frac{\cos(kx)}{x^2+a^2} dx$, where k > 0 and a > 0, is **[GATE-2019]**
(a) $\frac{\pi}{a} e^{-ka}$ (b) $\frac{2\pi}{a} e^{-ka}$ (c) $\frac{\pi}{2a} e^{-ka}$ (d) $\frac{3\pi}{2a} e^{-ka}$

TIFR- PREVIOUS YEAR QUESTIONS

58. If z = x+iy then the function *fI(x,y)* = (1+x+y) (1+x-y)+a(x^2-y^2) -1+2iy(1-x-ax) where *a* is a real parameter , is analytic in the complex z plane if *a* is equal to. **[TIFR 2013]**
(a) -1 (b) +1 (c) 0 (d) i

59. The integral $\int_0^{\infty} \frac{dx}{4+x^4}$ evaluates to. **[TIFR 2014]**
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

60. The integral $\int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta+a^2}$ where 0 < a <1, evaluates to. **[TIFR 2015]**
(a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

61. The value of the integral $\oint_C \frac{SinZ}{z^6} dz$, where C is the circle with centre z = 0 and radius 1 unit. **[TIFR 2016]**
(a) iπ (b) $\frac{i\pi}{120}$ (c) $\frac{i\pi}{60}$ (d) - $\frac{i\pi}{6}$

62. The value of the integral $\int_0^{\infty} \frac{dx}{x^4+4}$, is. **[TIFR 2017]**
(a) π (b) $\frac{\pi}{2}$ (c) - $\frac{\pi}{2}$ (d) $\frac{\pi}{8}$

63. The value of the integral $\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\cos x}{x^2+a^2}$ is. **[TIFR 2018]**
(a) 1/2*a* (b) 1/2π*a*
(c) π*a*exp(-*a*) (d) exp(-*a*)/*a*

64. Consider the complex function $f(x,y) = u(x,y) + iv(x,y)$ where $u(x,y) = x^2(2+x) - y^2(2+3x), v(x,y) = y(\lambda x + 3x^2 - y^2)$ and λ is real. If it is known that $f(x,y)$ is analytic in complex plane of z = $x + iy$, then it can be written. **[TIFR 2019]**

(a) $f = z^2 + z^3$ (b) $f = \bar{z}(2+\bar{z}^2)$
(c) $f = 2z\bar{z} + z^2 - \bar{z}^2$ (d) $f = z^2(2+z)$

JEST- PREVIOUS YEAR QUESTIONS

65. The value of integral $\int_0^\infty \frac{lnx}{(x^2+1)^2}, dx$ is. **[JEST-2012]**

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

66. Compute $\lim\limits_{z\to 0} \frac{Re(z^2)+Im(z^2)}{z^2}$. **[JEST-2013]**

(a) The limit does not exist (b) 1
(c) –i (d) -1

67. The value of integral. **[JEST-2014]**

$$I = \oint_c \frac{sinz}{2z-\pi} dz$$

With c a is circle $|z| = 2$, is

(a) 0 (b) 2πi (c) πi (d) –πi

68. The value of limit $\lim\limits_{z\to i} \frac{z^{10}+1}{z^6+1}$ is equal to. **[JEST-2014]**

(a) 1 (b) 0 (c) -10/3 (d) 5/3

69. Given an analytic function $f(x,y) = \phi(x,y) + i\psi(x,y)$ where $\phi(x,y) = x^3 + 4x - y^2 + 2y$. If C is a constant, then which of the following relation is true? **[JEST-2015]**

(a) $\psi(x,y) = x^2y + 4y + C$ (b) $\psi(x,y) = 2xy - 2x + C$
(c) $\psi(x,y) = 2xy + 4y - 2x + C$ (d) $\psi(x,y) = x^2y - 2x + C$

70. The value of the integral $\int_0^\infty \frac{\ln x}{(x^2+1)} dx$ is. **[JEST-2016]**

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (b)π^2 (d) 0

71. The sum of the infinite series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$ is **[JEST-2016]**

(a) 2π (b)π (c)$\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

72. The integral

$$\oint_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx \text{ is.}$$ **[JEST-2018]**

(a) $\frac{\pi}{e}$ (b) πe^{-2} (c) π (d)zero

73. Consider the function $f(x, y) = |x| - i|y|$. In which domain of the complex plane is this function analytic? **[JEST-2019]**
(a) First and second quadrants (b) Second and third quadrants
(c) Seconds and fourth quadrants (d) Nowhere

ANSWER-KEY

PART-A (MODULUS & ARGUMENT-CUBE ROOTS OF UNITY)

1.	A	2.	D	3.	D	4.	B	5.	B
6.	D	7.	B	8.	D	9.	C		

PART-B (COMPLEX FUNCTION & CAUCHY-REAMANN EQUATIONS)

1.	*	2.	*	3.	A	4.	A	5.	B
6.	B	7.	A	8.	B				

PART-C (MILNE THOMSON METHOD & ANALYTIC FUNCTION)

1.	*	2.	B	3.	A,B,C	4.	A,B,C,D	5.	D
6.	B	7.	B,D						

PART-D (POWER & TAYLOR SERIES EXPANSION)

1.	*	2.	B	3.	C	4.	*	5.	B
6.	C	7.	A	8.	B	9.	C	10.	A
11.	B	12.	*	13.	C				

PART-E (SINGULAR POINTS & CALCULATION OF RESIDUES)

1.	*	2.	D	3.	C	4.	D	5.	D
6.	C	7.	A	8.	C	9.	*	10.	C
11.	D	12.	B	13.	D	14.	*		

PART-F (APPLICATION OF CAUCHY RESIDUE THEOREM)

1.	B	2.	C	3.	A	4.	A	5.	D
6.	C	7.	D	8.	A	9.	D	10.	D
11.	*	12.	D	13.	*				

PART-G (IMPROPER INTEGRAL)

1.	*	2.	B	3.	D	4.	A	5.	*

PREVIOUS YEAR ANSWER-KEY

1	C	**13**	A	**25**	B	**37**	B	**49**	C	**61**	C	**73**	C
2	B	**14**	C	**26**	B	**38**	A	**50**	C	**62**	D		
3	B	**15**	B	**27**	D	**39**	B	**51**	B	**63**	D		
4	B	**16**	A	**28**	C	**40**	A	**52**	A	**64**	D		
5	B	**17**	B	**29**	C	**41**	B	**53**	Π	**65**	B		
6	A	**18**	C	**30**	A	**42**	C	**54**	3	**66**	C		
7	B	**19**	C	**31**	B	**43**	A	**55**	81.70	**67**	C		
8	A	**20**	D	**32**	B	**44**	C	**56**	*	**68**	D		
9	D	**21**	C	**33**	A	**45**	D	**57**	A	**69**	C		
10	A	**22**	C	**34**	D	**46**	B	**58**	A	**70**	D		
11	B	**23**	A	**35**	B	**47**	D	**59**	D	**71**	*		
12	C	**24**	*	**36**	C	**48**	3	**60**	A	**72**	A		